Your Name

Andrew ID

Honor Statement

I agree to complete this exam without unauthorized assistance from any person, materials, or device.

Signature: ________________________________

- Unless stated otherwise, you must show and justify all of your work to receive credit.
- If you use scratch paper that you want graded, write your name on it and leave a note on the original page of the problem.
- The exam is graded out of 100 points.

Question	Points	Score
1	20	
2	20	
3	20	
4	20	
5	20	
Total	100	

Problem 1. (20 pts)
Let F, G, H be three subspaces of a vector space E. Is it necessarily true that

$$H \cap (F + G) \;=\; (H \cap F) + (H \cap G)\,?$$

Justify your answer (prove it or give a counterexample).

Problem 2. (20 pts)
Let $E = \mathbb{R}^{\mathbb{R}}$ be the vector space of all real-valued functions on $\mathbb{R}$. Define

$$F = \{f \in E : \ f(0) + f(1) = 0\}, \qquad G = \{f \in E : \ f \text{ is constant on } \mathbb{R}\}.$$

1. Prove that F and G are vector subspaces of E.

2. Prove that F and G are complementary subspaces in E, i.e. that

$$E = F \oplus G.$$

Problem 3. (20 pts)
Let $n \in \mathbb{N}$. For each $k \in \{0, 1, \ldots, n\}$, define the function

$$f_k : \mathbb{R} \to \mathbb{R}, \qquad f_k(x) = e^{kx}.$$

Show that the family $(f_k)_{0 \leq k \leq n}$ is linearly independent in the vector space $F(\mathbb{R}, \mathbb{R})$ of all real-valued functions on $\mathbb{R}$.

Problem 4. (20 pts)
Let V be a finite-dimensional vector space, and let $V_1, V_2 \subseteq V$ be subspaces. Prove that

$$\dim(V_1 + V_2) = \dim(V_1) + \dim(V_2) - \dim(V_1 \cap V_2).$$

You may use **without** proof the fact that any linearly independent list in a finite-dimensional vector space can be extended to a basis.

Problem 5. (20 pts) Let $E = \mathbb{R}^4$ and $F = \mathbb{R}^2$ be real vector spaces. Consider

$$H = \{(x,y,z,t) \in \mathbb{R}^4 \,; x = y = z = t\}.$$

Do there exist linear maps from E to F whose kernel is H?