Linear Algebra (21-341) — Spring 2026

Carnegie Mellon University

Lectures: MWF 4:00–4:50 PM
Location: Hamerschlag Hall (HH) B131, Pittsburgh, PA
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Course Website: Canvas
Office Hours: TBA

Course Description

This course presents the fundamental notions of linear and bilinear algebra, emphasizing abstract structure and conceptual understanding. Central topics include vector spaces and linear maps, determinants, inner products, quadratic forms, and spectral theory. Particular attention is given to the reduction of linear operators and matrices, especially in the real symmetric case, and to the role of orthogonal projections and their applications. The course balances rigorous reasoning with problem-solving and prepares students for further work in mathematics and related fields.

Learning Objectives

By the end of the course, students will be able to:

- Develop a solid understanding of the abstract structure of vector spaces and linear maps, and work fluently with bases, coordinates, and duality.
- Master core objects of linear algebra, including determinants, inner products, quadratic forms, and spectral theory, and understand their role in the structure of linear operators.
- Strengthen proof-writing skills and conceptual reasoning, with an emphasis on abstraction, while also appreciating key applications.

Course Content (Planned)

The following outline describes the planned structure of the course. The pace and emphasis may be adjusted depending on the progress of the class.

1. **Vector Spaces and Linear Maps**
   Vector spaces and subspaces; linear independence, spanning families, bases, and dimension. Kernel, image, and rank of a linear map. Direct sums and associated projections.

2. **Matrices and Change of Basis**
   Matrix representation of linear maps. Change of basis. Matrix operations. Equivalent and similar matrices and their characterizations.

3. **Determinants and Invertibility**
   Determinants of families of vectors, linear maps, and square matrices. Properties and computation. Cofactor expansion and matrix inversion.

4. **Linear Systems and Elementary Operations**
   Elementary row and column operations. Rank computation, matrix inversion, and solution of linear systems.

5. **Spectral Theory and Reduction of Linear Operators**
   Eigenvalues and eigenvectors. Characteristic polynomial. Diagonalization of linear operators and matrices. Criteria for diagonalizability. Spectral theorem.

6. **Applications of Matrix Reduction**
   Applications to matrix powers and linear recurrence relations.

7. **Quadratic Forms**
   Bilinear and quadratic forms. Associated matrices and change of basis. Congruent matrices. Gauss reduction. Orthogonal bases and signature.

8. **Euclidean Spaces and Orthogonality**
   Inner product spaces. Gram–Schmidt orthogonalization process. Orthogonal matrices and their properties. Orthogonal projections and distance to subspaces.

9. **Real Symmetric Matrices**
   Real symmetric matrices and self-adjoint operators. Diagonalization in orthonormal bases. Simultaneous reductions.

Literature

There is no single required textbook for this course. Lecture notes will constitute the primary reference.

Recommended references:

- Serge Lang, *Linear Algebra*, 3rd Edition. Springer. A classic reference for a rigorous and conceptual treatment.
- Sheldon Axler, *Linear Algebra Done Right*, 4th Edition. A widely used and accessible reference emphasizing linear maps and structure.

Course Format

This is an in-person course. Active participation is expected, including asking and answering questions and engaging in class discussions.

Homework

There will be approximately **10 homework assignments** during the semester, submitted via **Gradescope**. Late submissions will not be accepted. The **lowest two homework scores will be dropped** when computing the final grade.

Homework Policy and Academic Integrity

You are strongly encouraged to work with classmates on homework assignments. Discussing problems and sharing ideas is an excellent way to learn. However, all submitted solutions must be written independently, in your own words and style.

You may consult external resources (such as additional textbooks, online notes, or electronic tools including ChatGPT or similar AI systems), provided that you clearly acknowledge any such use and indicate where assistance was obtained. Using external resources without attribution may be considered academic misconduct. Most importantly, you are responsible for understanding and being able to reproduce any argument you submit.

Students are strongly encouraged to make use of **office hours**, which are an essential part of the course. Office hours provide a dedicated space to ask questions, clarify concepts, and discuss problem-solving strategies in a collaborative and supportive environment.

Exams and Grading

Exams. There will be **three in-class midterm exams** during the semester, based on lecture material and homework. There will be **no final exam**.

Grading. The final course grade will be computed as follows:

- 60% Midterm Exams
- 40% Homework

The midterm grade will be based solely on the first midterm exam. Standard letter-grade cutoffs will be used (A, B, C, D at 90, 80, 70, 60, respectively).